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Higher dimensional flat embeddings of (2+1) dimensional black holes

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Abstract

We obtain the higher dimensional global flat embeddings of static, rotating, and charged BTZ black holes. On the other hand, we also study the similar higher dimensional flat embeddings of the (2+1) de Sitter black holes which are the counterparts of the anti-de Sitter BTZ black holes. As a result, the charged dS black hole is shown to be embedded in (3+2) GEMS, contrast to the charged BTZ one having (3+3) GEMS structure.

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I. INTRODUCTION

After Unruh's work [1], it has been known that a thermal Hawking effect on a curved manifold [2] can be looked at as an Unruh effect in a higher flat dimensional space time. According to the global embedding Minkowski space (GEMS) approach [3–7], several authors [8–11] recently have shown that this approach could yield a unified derivation of temperature for various curved manifolds such as rotating Banados-Teitelboim-Zanelli (BTZ) [12–16], Schwarzschild [17] together with its anti-de Sitter (AdS) extension, Reissner-Nordström (RN) [18,19] and RN-AdS [20].

On the other hand, since its pioneering work in 1992, the (2+1) dimensional BTZ black hole [12,13] has become one of useful models for realistic black hole physics [14]. Moreover, significant interests in this model have recently increased with the novel discovery that the thermodynamics of higher dimensional black holes can often be interpreted in terms of the BTZ solution [21]. It is therefore interesting to study the geometry of (2+1) dimensional black holes and their thermodynamics through further investigation.

In this paper we will analyze Hawking and Unruh effects of the (2+1) dimensional black holes in terms of the GEMS approach. In section 2 after we briefly recapitulate the known global (2+2) dimensional embedding of the static and rotating (2+1) BTZ black holes, we will newly consider the charged static BTZ black hole. In section 3, we will also treat the novel global higher dimensional flat embeddings of the (2+1) static, rotating, and charged de Sitter(dS) black holes, which are the counterpart of usual BTZ black holes. In particular, we will show that the charged dS black hole is embedded in (3+2) GEMS, contrast to the charged BTZ one with (3+3) GEMS structure.

II. BTZ ANTI-DE SITTER GEOMETRY

A. Static BTZ Space

In this subsection we begin with a brief recapitulation of the GEMS approach to temperature given in Ref. [13], for the well known (2+1) dimensional uncharged static BTZ black hole which is described by the 3-metric

$$ds^2 = N^2 dt^2 - N^{-2} dr^2 - r^2 d\phi^2 \tag{1}$$

with the lapse function

$$N^2 = -M + \frac{r^2}{l^2}. \tag{2}$$

Here one notes that this BTZ space originates from AdS one via the geodesic identification $\phi = \phi + 2\pi$. The (2+2) AdS GEMS $ds^2 = (dz^0)^2 - (dz^1)^2 - (dz^2)^2 + (dz^3)^2$ is then given by the coordinate transformations for $r \geq r_H$ (the extension to $r < r_H$ is given in Ref. [13].) with the event horizon $r_H = M^{1/2} l$ as follows

$$\begin{aligned}
z^0 &= k_H^{-1}\left(\frac{r^2 - r_H^2}{l^2}\right)^{1/2} \sinh \frac{r_H}{l^2} t, \\
z^1 &= k_H^{-1}\left(\frac{r^2 - r_H^2}{l^2}\right)^{1/2} \cosh \frac{r_H}{l^2} t, \\
z^2 &= l \frac{r}{r_H} \sinh \frac{r_H}{l} \phi, \\
z^3 &= l \frac{r}{r_H} \cosh \frac{r_H}{l} \phi,
\end{aligned} \tag{3}$$

where $k_H = r_H / l^2$ is the Hawking-Bekenstein horizon surface gravity. These flat embeddings of the curved spacetime are easily obtained by comparing the 3-metric (1) with $ds^2 = \eta_{ab} dz^a dz^b$, where $a, b = 0, \cdots, 3$ and $\eta_{ab} = (+, -, -, +)$.

In static detectors (ϕ, $r =$ const) described by a fixed point in the (z^2, z^3) plane (for example $\phi = 0$ gives $z^2 = 0$, $z^3 =$const), one can have constant 3-acceleration

$$a = \frac{r}{l(r^2 - r_H^2)^{1/2}} \tag{4}$$

and constantly accelerated motion in (z^0,z^1) with the Hawking temperature

$$T = \frac{a_4}{2\pi} = \frac{r_H}{2\pi l(r^2 - r_H^2)^{1/2}} \tag{5}$$

which yields the relation $a_4 = (a^2 - l^{-2})^{1/2}$. Here one notes that the above Hawking temperature is also given by the relation [2,23,24]

$$T = \frac{1}{2\pi}\frac{k_H}{g_{00}^{1/2}}. \tag{6}$$

Note that in the asymptotic limit the BTZ space approaches to AdS one whose acceleration at infinity is given by $a = l^{-1}$ to yield zero temperature (no Hawking particle at infinity). The Rindler horizon condition $(z^1)^2-(z^0)^2 = 0$ implies $r = r_H$ and the remaining embedding constraint yields $(z^3)^2 - (z^2)^2 = l^2$ so that the BTZ solution yields a finite Unruh area $2\pi r_H$ due to the periodic identification of ϕ mod 2π [22]. The well-known entropy $2\pi r_H$ of the static BTZ space is then given by the transverse Rindler area [25].

B. Rotating BTZ Space

In this subsection we also briefly summarize the results of the GEMS approach given in Ref. [13,14,22], for the well known (2+1) dimensional uncharged rotating BTZ black hole which is described by the 3-metric

$$ds^2 = N^2dt^2 - N^{-2}dr^2 - r^2(d\phi + N^\phi dt)^2, \tag{7}$$

where the lapse and shift functions are

$$N^2 = -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2}, \quad N^\phi = -\frac{J}{2r^2}, \tag{8}$$

respectively. Note that for the nonextremal case there exist two horizons $r_\pm(J)$ satisfying the following equations,

$$0 = -M + \frac{r_\pm^2}{l^2} + \frac{J^2}{4r_\pm^2}, \tag{9}$$

respectively. Then, without solving these equations explicitly we can rewrite the mass M and angular momentum J in terms of these outer and inner horizons as follows

$$M = \frac{r_+^2 + r_-^2}{l^2}, \quad J = \frac{2r_+ r_-}{l}. \tag{10}$$

Furthermore by using these relations, we can rewrite the lapse and shift functions as

$$N^2 = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2 l^2}, \quad N^\phi = -\frac{r_+ r_-}{r^2 l}, \tag{11}$$

respectively. Here one notes that this BTZ space originates from AdS one via the geodesic identification $\phi = \phi + 2\pi$. The (2+2) AdS GEMS $ds^2 = (dz^0)^2 - (dz^1)^2 - (dz^2)^2 + (dz^3)^2$ is then given by the coordinate transformations for $r \geq r_+$ as follows

$$\begin{aligned}
z^0 &= k_H^{-1} \left(\frac{(r^2 - r_+^2)(r_+^2 - r_-^2)}{r_+^2 l^2} \right)^{1/2} \sinh \left(\frac{r_+}{l^2} t - \frac{r_-}{l} \phi \right), \\
z^1 &= k_H^{-1} \left(\frac{(r^2 - r_+^2)(r_+^2 - r_-^2)}{r_+^2 l^2} \right)^{1/2} \cosh \left(\frac{r_+}{l^2} t - \frac{r_-}{l} \phi \right), \\
z^2 &= l \left(\frac{r^2 - r_-^2}{r_+^2 - r_-^2} \right)^{1/2} \sinh \left(\frac{r_+}{l} \phi - \frac{r_-}{l^2} t \right), \\
z^3 &= l \left(\frac{r^2 - r_-^2}{r_+^2 - r_-^2} \right)^{1/2} \cosh \left(\frac{r_+}{l} \phi - \frac{r_-}{l^2} t \right),
\end{aligned} \tag{12}$$

where the surface gravity is given as $k_H = (r_+^2 - r_-^2)/(r_+ l^2)$. For the trajectories, which follow the Killing vector $\xi = \partial_t - N^\phi \partial_\phi$, one can obtain constant 3-acceleration [22]

$$a = \frac{r^4 - r_+^2 r_-^2}{r^2 l (r^2 - r_+^2)^{1/2} (r^2 - r_-^2)^{1/2}}. \tag{13}$$

and the Hawking temperature [14,26]

$$T = \frac{a_4}{2\pi} = \frac{r(r_+^2 - r_-^2)}{2\pi r_+ l (r^2 - r_+^2)^{1/2} (r^2 - r_-^2)^{1/2}}. \tag{14}$$

Here these trajectories do not describe pure Rindler motion in the GEMS combining accelerated motion in the (z^0, z^1) plane with a spacelike motion in (z^2, z^3) [22].

Finally, the entropy of the rotating BTZ space is given by $2\pi r_+(J)$ which reproduces the uncharged static BTZ black hole entropy $2\pi r_H$ in the vanishing J limit. Note that all results in this subsection will be useful to analyze the dS cases.

C. Charged BTZ Space

We now newly consider the charged static BTZ black hole solution where the 3-metric (1) is described by the charged lapse [12,27]

$$N^2 = -M + \frac{r^2}{l^2} - 2Q^2 \ln r. \tag{15}$$

Here the mass M can be rewritten as $M = \frac{r_H^2}{l^2} - 2Q^2 \ln r_H$ with the horizon $r_H(Q)$, which is the root of $-M + \frac{r^2}{l^2} - 2Q^2 \ln r = 0$.

As in the previous two cases, we can first find the $-r^2 d\phi^2$ term in the 3-metric by introducing two coordinates (z^3, z^4) in Eq. (19), giving $-(dz^3)^2 + (dz^4)^2 = -r^2 d\phi^2 + \frac{l^2}{r_H^2} dr^2$. Then, in order to obtain the $N^2 dt^2$ term, we make ansatz of two coordinates, (z^0, z^1) in Eq. (19), which, together with the above (z^3, z^4), yields

$$\begin{aligned}
&(dz^0)^2 - (dz^1)^2 - (dz^3)^2 + (dz^4)^2 \\
&= N^2 dt^2 - \left(\frac{r_H^2 (\frac{r^2}{l^2} - \frac{Q^2 r_H}{r})^2}{(\frac{r_H^2}{l^2} - Q^2)^2 (\frac{r^2 - r_H^2}{l^2} - 2Q^2 \ln \frac{r}{r_H})} - \frac{l^2}{r_H^2} \right) dr^2 - r^2 d\phi^2.
\end{aligned} \tag{16}$$

Since the combination of $N^{-2} dr^2$ and dr^2 term in Eq. (16) can be separated into positive definite part and negative one as follows

$$\begin{aligned}
&\left(k_H^{-1} Q^2 \frac{l[r^2 + r_H^2 + 2r^2 f(r, r_H)]^{1/2}}{r_H^2 r [1 - \frac{Q^2 l^2}{r_H^2} f(r, r_H)]^{1/2}} \mathrm{d}r \right)^2 - \left(k_H^{-1} Q \frac{[2r_H^2 + \frac{r_H^4 + Q^4 l^4}{r_H^2} f(r, r_H)]^{1/2}}{r_H^2 [1 - \frac{Q^2 l^2}{r_H^2} f(r, r_H)]^{1/2}} \mathrm{d}r \right)^2 \\
&\equiv (dz^2)^2 - (dz^5)^2,
\end{aligned} \tag{17}$$

we can obtain desired flat global embeddings of the corresponding curved 3-metric as

$$\begin{aligned}
ds^2 &= (dz^0)^2 - (dz^1)^2 - (dz^2)^2 - (dz^3)^2 + (dz^4)^2 + (dz^5)^2 \\
&= N^2 dt^2 - N^{-2} dr^2 - r^2 d\phi^2.
\end{aligned} \tag{18}$$

Note that the z^2 and z^5 are monotonic functions in the range of $Ql \le r_H < r$.

As results, we here summarize the (3+3) AdS GEMS given by the following coordinate transformations with an additional timelike dimension z^5

$$
\begin{aligned}
z^0 &= k_H^{-1}\left(\frac{r^2-r_H^2}{l^2}-2Q^2\ln\frac{r}{r_H}\right)^{1/2}\sinh k_H t\\
z^1 &= k_H^{-1}\left(\frac{r^2-r_H^2}{l^2}-2Q^2\ln\frac{r}{r_H}\right)^{1/2}\cosh k_H t\\
z^2 &= k_H^{-1}Q^2\int \mathrm{d}r\frac{l[r^2+r_H^2+2r^2 f(r,r_H)]^{1/2}}{r_H^2 r[1-\frac{Q^2l^2}{r_H^2}f(r,r_H)]^{1/2}}\\
z^3 &= l\frac{r}{r_H}\sinh\frac{r_H}{l}\phi\\
z^4 &= l\frac{r}{r_H}\cosh\frac{r_H}{l}\phi\\
z^5 &= k_H^{-1}Q\int \mathrm{d}r\frac{[2r_H^2+\frac{r_H^4+Q^4l^4}{r_H^2}f(r,r_H)]^{1/2}}{r_H^2[1-\frac{Q^2l^2}{r_H^2}f(r,r_H)]^{1/2}},
\end{aligned}
\tag{19}
$$

where the surface gravity is given by $k_H=[(r_H/l)^2-Q^2]/r_H$ and

$$
f(r,r_H)=\frac{2r_H^2}{r^2-r_H^2}\ln\frac{r}{r_H} \tag{20}
$$

which, due to L'Hospital's rule, approaches to unity as r goes to infinity.

In static detectors (ϕ, r = const) described by a fixed point in the (z^2,z^3) plane (for example $\phi=0$ gives $z^2=0$, z^3 =const), one can have constant 3-acceleration

$$
a=\frac{r-Q^2l^2/r}{l[r^2-r_H^2-2Q^2l^2\ln(r/r_H)]^{1/2}} \tag{21}
$$

and constantly accelerated motion in (z^0,z^1) with the Hawking temperature

$$
T=\frac{a_6}{2\pi}=\frac{r_H-Q^2l^2/r_H}{2\pi l[r^2-r_H^2-2Q^2l^2\ln(r/r_H)]^{1/2}}, \tag{22}
$$

which is also attainable via the relation (6). Note that, in the GEMS where one can have a constant Rindler-like accelerated motion, the temperature (22) measured by the detector agrees with the temperature given by the response function of particle detectors [28]. Here one can easily check that, in the uncharged limit where the spacelike z^2 and timelike z^5 dimensions in Eq. (19) vanish, the above coordinate transformations are exactly reduced to the previous one (3) for the uncharged static BTZ case. Moreover, the desired black hole temperature is given as

$$
T_0=g_{00}^{1/2}T=\frac{(r_H/l)^2-Q^2}{2\pi r_H}, \tag{23}
$$

which enters into the black hole thermodynamics relations. Here one notes that use of incomplete embedding spaces, that cover only $r > r_H$ (as for example in Ref. [6]), will lead to observers there for whom there is no event horizon, no loss of information, and no temperature.

We now see how the BTZ solution yields a finite Unruh area due to the periodic identification of ϕ mod 2π. The Rindler horizon condition $(z^1)^2 - (z^0)^2 = 0$ implies $r = r_H$ and the remaining embedding constraints yield $z^2 = f_1(r)$, $z^5 = f_2(r)$ and $(z^4)^2 - (z^3)^2 = l^2$ where $f_1(r)$ and $f_2(r)$ can be read off from Eq. (19). The area of the Rindler horizon is now described as

$$\int \mathrm{d}z^2 \mathrm{d}z^3 \mathrm{d}z^4 \mathrm{d}z^5 \delta(z^2 - f_1(r))\delta(z^5 - f_2(r))\delta([(z^4)^2 - (z^3)^2]^{1/2} - l)$$

which, after performing trivial integrations over z^2 and z^5, yields the desired entropy of the charged BTZ space

$$\begin{aligned} &\int_{-l\sinh(\pi r_H/l)}^{l\sinh(\pi r_H/l)} \mathrm{d}z^3 \int_0^{[(z^3)^2+l^2]^{1/2}} \mathrm{d}z^4 \delta([(z^4)^2 - (z^3)^2]^{1/2} - l) \\ &= \int_{-l\sinh(\pi r_H/l)}^{l\sinh(\pi r_H/l)} \mathrm{d}z^3 \frac{l}{[l^2 + (z^3)^2]^{1/2}} = 2\pi r_H(Q), \end{aligned} \tag{24}$$

which reproduces the entropy $2\pi r_H$ of the uncharged BTZ case in the limit $Q \to 0$.

It seems appropriate to comment on the minimal extra dimensions needed for desired GEMS. As you may know, spaces of constant curvature can be embedded into flat space with only single extra dimension. This is seen in the previous subsections for the static and rotating BTZ cases, which are embedded in (2+2)-dimensional spaces. On the other hand, since the charged BTZ solution is not locally AdS, we have introduced three extra dimensions for desired GEMS. In the next section, we will also obtain similar results for the uncharged and charged (2+1)-dimensional dS cases.

III. (2+1) DE SITTER BLACK HOLES

A. Static de Sitter Space

The static dS black hole [29,30] is described by the 3-metric (1) with the lapse function

$$N^2 = M - \frac{r^2}{l^2}. \tag{25}$$

It arises from dS upon making the geodesic identification $\phi = \phi + 2\pi$. The coordinate transformations to the (3+1) dS GEMS $ds^2 = (dz^0)^2 - (dz^1)^2 - (dz^2)^2 - (dz^3)^2$ are for $r \leq r_H$ with the event horizon $r_H = M^{1/2} l$

$$\begin{aligned} z^0 &= k_H^{-1}\left(\frac{r_H^2 - r^2}{l^2}\right)^{1/2} \sinh \frac{r_H}{l^2} t, \\ z^1 &= k_H^{-1}\left(\frac{r_H^2 - r^2}{l^2}\right)^{1/2} \cosh \frac{r_H}{l^2} t, \\ z^2 &= l \frac{r}{r_H} \sin \frac{r_H}{l} \phi, \\ z^3 &= l \frac{r}{r_H} \cos \frac{r_H}{l} \phi, \end{aligned} \tag{26}$$

where the constant r_H are related to the mass.

Even though there is no longer a one to one mapping between the dS GEMS and the BTZ like dS space due to the ϕ identification, following a detector motion with certain initial condition such as $\phi(t = 0) = 0$ still yields a unique trajectory in the embedding space. If the detector trajectory maps into an Unruh one in the dS GEMS without ambiguity, then one can exploit it to evaluate temperature.

Now let us consider static detectors (ϕ, r = const). These detectors have constant 3-acceleration

$$a = \frac{r}{l(r_H^2 - r^2)^{1/2}}, \tag{27}$$

and are described by a fixed point in the (z^2, z^3) plane (for example $\phi = 0$ gives $z^2 = 0$, z^3 =const), to yield constantly accelerated motion in (z^0, z^1) with the Hawking temperature

$$T = \frac{a_4}{2\pi} = \frac{r_H}{2\pi l(r_H^2 - r^2)^{1/2}}, \tag{28}$$

which is connected with a by the relation $a_4 = (a^2 + l^{-2})^{1/2}$. Thus, in the GEMS described in (26), we have a constant Rindler-like accelerated motion and the temperature (28) measured by the detector, which agrees with the temperature given by the response function of particle detectors [28]. We also obtain the entropy $2\pi r_H$ of the static dS space as in the BTZ case.

B. Rotating de Sitter Space

The rotating Kerr-dS black hole [25,31] is described by the 3-metric (7) with the lapse and shift functions

$$N^2 = M - \frac{r^2}{l^2} + \frac{J^2}{4r^2}, \quad N^\phi = -\frac{J}{2r^2}. \tag{29}$$

Similarly to the rotating BTZ black hole case, we can also rewrite the mass M and angular momentum J in terms of outer and inner horizons $r_\pm(J)$ as follows

$$M = \frac{r_+^2 - r_-^2}{l^2}, \quad J = \frac{2r_+ r_-}{l}. \tag{30}$$

Furthermore, by using these relations, we can obtain the lapse and shift functions

$$N^2 = \frac{(r_+^2 - r^2)(r^2 + r_-^2)}{r^2 l^2}, \quad N^\phi = -\frac{r_+ r_-}{r^2 l}, \tag{31}$$

respectively. It arises from dS upon making the geodesic identification $\phi = \phi + 2\pi$. The coordinate transformations to the (3+1) dS GEMS $ds^2 = (dz^0)^2 - (dz^1)^2 - (dz^2)^2 - (dz^3)^2$ are for $r \leq r_+$

$$\begin{aligned}
z^0 &= k_H^{-1} \left(\frac{(r_+^2 + r_-^2)(r_+^2 - r^2)}{r_+^2 l^2} \right)^{1/2} \sinh\left(\frac{r_+}{l^2} t - \frac{r_-}{l}\phi \right), \\
z^1 &= k_H^{-1} \left(\frac{(r_+^2 + r_-^2)(r_+^2 - r^2)}{r_+^2 l^2} \right)^{1/2} \cosh\left(\frac{r_+}{l^2} t - \frac{r_-}{l}\phi \right), \\
z^2 &= l \left(\frac{r^2 + r_-^2}{r_+^2 + r_-^2} \right)^{1/2} \sin\left(\frac{r_+}{l}\phi + \frac{r_-}{l^2} t \right), \\
z^3 &= l \left(\frac{r^2 + r_-^2}{r_+^2 + r_-^2} \right)^{1/2} \cos\left(\frac{r_+}{l}\phi + \frac{r_-}{l^2} t \right),
\end{aligned} \tag{32}$$

where the constants $r_\pm(J)$ are related to the mass and angular momentum as in Eq. (30).

Similar to the rotating BTZ case, for the trajectories which follow the Killing vector $\xi = \partial_t - N^\phi \partial_\phi$, we obtain constant 3-acceleration

$$a = \frac{r^4 + r_+^2 r_-^2}{r^2 l (r_+^2 - r^2)^{1/2} (r^2 + r_-^2)^{1/2}}, \tag{33}$$

and the Hawking temperature

$$T = \frac{a_4}{2\pi} = \frac{r(r_+^2 + r_-^2)}{2\pi r_+ l (r_+^2 - r^2)^{1/2} (r^2 + r_-^2)^{1/2}}. \tag{34}$$

Note that as in the rotating BTZ black hole these trajectories do not describe pure Rindler motion in the GEMS. On the other hand, the entropy $2\pi r_+(J)$ of the rotating Kerr-dS space also reproduces the uncharged static dS black hole entropy $2\pi r_H$ in the vanishing J limit.

C. Charged de Sitter Space

We now consider the charged static dS black hole solution where the 3-metric (1) is described by the charged lapse

$$N^2 = M - \frac{r^2}{l^2} - 2Q^2 \ln r. \tag{35}$$

Here the mass M can be rewritten as $M = \frac{r_H^2}{l^2} + 2Q^2 \ln r_H$ with the horizon $r_H(Q)$, which is the root of $M - \frac{r^2}{l^2} - 2Q^2 \ln r = 0$.

After similar algebraic manipulation by following the previous steps described in Sec. II.C, we obtain the (3+2) dS GEMS $ds^2 = (dz^0)^2 - (dz^1)^2 - (dz^2)^2 - (dz^3)^2 + (dz^4)^2$ given by the coordinate transformations with only one additional timelike dimension z^4, in contrast to the BTZ case where one needs to require additionally one spacelike and one timelike dimensions

$$\begin{aligned}
z^0 &= k_H^{-1} \left(\frac{r_H^2 - r^2}{l^2} - 2Q^2 \ln \frac{r}{r_H} \right)^{1/2} \sinh k_H t, \\
z^1 &= k_H^{-1} \left(\frac{r_H^2 - r^2}{l^2} - 2Q^2 \ln \frac{r}{r_H} \right)^{1/2} \cosh k_H t, \\
z^2 &= l \frac{r}{r_H} \sin \frac{r_H}{l} \phi,
\end{aligned}$$

$$z^3 = l\frac{r}{r_H}\cos\frac{r_H}{l}\phi,$$

$$z^4 = k_H^{-1}Q\int \mathrm{d}r\frac{\{Q^2l^2[r^2+r_H^2+2r^2f(r,r_H)]+r^2[2r_H^2+\frac{r_H^4+Q^4l^4}{r_H^2}f(r,r_H)]\}^{1/2}}{r_H^2 r(1+\frac{Q^2l^2}{r_H^2}f(r,r_H))^{1/2}},\tag{36}$$

where the surface gravity is given by $k_H = [(r_H/l)^2+Q^2]/r_H$ and $f(r,r_H)$ is given by Eq. (20). Here one can easily check that, in the uncharged limit, the above coordinate transformations are reduced to the previous one (26) for the uncharged static dS case. Note that in dS space we need only one additional dimension z^4 since the Q^2 term in the numerator has the same sign with respect to the second term, differently from the charged static BTZ case where we have the opposite relative sign between these two terms to yield two additional dimensions, namely the spacelike z^2 and timelike z^5 in (19).

In static detectors (ϕ, $r =$ const) described by a fixed point in the (z^2, z^3) plane (for example $\phi = 0$ gives $z^2 = 0$, z^3 =const), one can have constant 3-acceleration

$$a = \frac{r+Q^2l^2/r}{l[r_H^2-r^2-2Q^2l^2\ln(r/r_H)]^{1/2}}\tag{37}$$

and the Hawking temperature in constantly accelerated motion in (z^0,z^1)

$$T = \frac{a_5}{2\pi} = \frac{r_H+Q^2l^2/r_H}{2\pi l[r_H-r^2-2Q^2l^2\ln(r/r_H)]^{1/2}}.\tag{38}$$

In the GEMS one can thus have a constant Rindler-like accelerated motion and the above Hawking temperature measured by the detector. Note that, in the uncharged static limit $Q\to 0$, the above 3-acceleration and Hawking temperature are reduced to the previous ones (27) and (28). The desired black hole temperature is then given as

$$T_0 = g_{00}^{1/2}T = \frac{(r_H/l)^2+Q^2}{2\pi r_H}.\tag{39}$$

Note that the entropy $2\pi r_H(Q)$ of the charged dS black hole is also given by the Rindler horizon condition. This is also reduced to the entropy $2\pi r_H$ for the uncharged static dS case in the limit of $Q\to 0$.

IV. CONCLUSION

In conclusion, we have shown that Hawking thermal properties map into their Unruh equivalents by globally embedding various curved (2+1) dimensional BTZ and dS spaces into higher dimensional flat ones. The relevant curved space detectors become Rindler ones, whose temperature and entropy reproduce the originals. It would be interesting to consider other interesting applications of GEMS, for example to superradiance in rotating Kerr type geometries [12,15,32,33] or Chan's new classes of static BTZ black hole solution due to a chosen asymptotically constant dilation and scalar [34].

Finally, it seems appropriate to comment on the rotating version of charged BTZ black hole. As pointed out by several authors, if one includes electric charge Q, the solution of the field equations is attainable only when the angular momentum J vanishes [14,35]. However, very recently several authors [36] have analyzed the case when all three 'hairs' M, J, and Q are different from zero although they have treated in some restricted ranges. Therefore, it is very interesting to show whether the solutions of the rotating charged BTZ and dS black holes may be obtained or not in terms of GEMS approach through further investigation.

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